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Flavor symmetry breaking and mixing effects in the $\eta\gamma$ and $\eta'\gamma$ transition form factors

T. Feldmann, P. Kroll

Department of Theoretical Physics, University of Wuppertal, D-42097 Wuppertal, Germany
(e-mail: kroll@theorie.physik.uni-wuppertal.de)

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Abstract. We reanalyze the $\eta\gamma$ and $\eta'\gamma$ transition form factors within the modified hard scattering approach on the basis of new experimental data from CLEO [1] and L3 [2]. Our approach perfectly describes the experimental data over a wide range of the virtuality of the probing photon, 1 GeV$^2 \leq Q^2 \leq$ 15 GeV^2. The analysis provides hints that the conventional flavor octet-singlet scheme for the η-η' mixing is too simple. A more general mixing scheme on the other hand, involving two mixing angles, leads to a very good description of the transition form factors and also accounts for the two-photon decay widths of the η and η' mesons as well as for the ratio of the widths for the $J/\psi \to \eta'\gamma$ and $J/\psi \to \eta\gamma$ decays. We also investigate the questions of possible deviations of the η and η' distribution amplitudes from the asymptotic form and of eventual intrinsic charm in the η and η' mesons. We estimate the charm decay constant of the η' meson to lie within the range -65 MeV $\leq f^c_{\eta'} \leq 15$ MeV.

1 Introduction

In 1995 the CLEO collaboration has presented their preliminary data on pseudoscalar meson-photon transition form factors (see Fig. 1) at large momentum transfer, Q^2, for the first time [3]. Since then these form factors attracted the interest of many theoreticians, and it can be said now that the CLEO measurement has strongly stimulated the field of hard exclusive reactions. One of the exciting aspects of the $\pi\gamma$ form factor is that it possesses a well-established asymptotic behavior [4,5], namely $F_{\pi\gamma} \to \sqrt{2} f_\pi/Q^2$ where $f_\pi(= 131\,\text{MeV})$ is the decay constant of the pion[1]. At the upper end of the measured momentum transfer range the CLEO data [3,1] only deviate by about 15% from that limiting value. Many theoretical papers are devoted to the explanation of that little difference. The perhaps most important outcome of these analyses, as far as they are based upon perturbative approaches (see e.g. [7–9]), is the rather precise determination of the pion's light-cone wave function. It turns out that the pion's distribution amplitude, i.e. its wave function integrated over transverse momentum, is close to the asymptotic form. This result has far-reaching consequences for the explanation of many hard exclusive reactions in which pions participate (see, for instance, [10,11]).

The situation is much more complicated for the $\eta\gamma$ and $\eta'\gamma$ transition form factors than in the $\pi\gamma$ case. In general there are at least four independent wave functions associated with the η and η' valence Fock states, since one has to consider $SU(3)_F$-singlet (octet) admixtures to the $\eta(\eta')$ mesons. Moreover, on account of the $U(1)_A$ anomaly, there is also the possibility of intrinsic charm and gluon admixtures. Correspondingly numerous are the decay constants being related to the configuration space wave functions at the origin. A full-fledged analysis of the transition form factors, taking into account all these components of the η and η' mesons, is beyond feasibility. Although the recent large momentum transfer data on the $\eta\gamma$ and $\eta'\gamma$ transition form factors measured by CLEO [1] and L3 [2] allow a more refined analysis of these processes than it was possible previously [8,12], additional phenomenological constraints as well as simplifying assumptions on the wave functions and the decay constants are still required. Thus, for instance, we will use the two-photon decays of the η and η' mesons as a constraint. The chiral anomaly combined with the PCAC hypothesis relates the decay constants to the decay widths for these two processes. Another constraint is offered by the ratio $R_{J/\psi}$ of the $J/\psi \to \eta'\gamma$ and $J/\psi \to \eta\gamma$ decay widths, since it can also be expressed in terms of the decay constants.

An obvious possibility to reduce the degrees of freedom in the analysis is the use of the conventional $SU(3)_F$ octet-

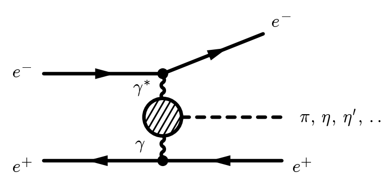

Fig. 1. Meson-photon transition form factors in e^+e^- collisions

[1] For a critical discussion of the asymptotic behavior of $F_{\pi\gamma}$ see e.g. [6] and references therein

singlet mixing scheme. Leaving aside eventual gluon and charm components, only two independent wave functions remain thereby and, hence, only two decay constants. The relative strength of the singlet and octet components in the physical mesons is then controlled by the pseudoscalar mixing angle θ_P. This mixing scheme has been used in previous analyses of the $\eta\gamma$, $\eta'\gamma$ transition form factors throughout [8,12,13]. As it will turn out from our analysis of the new large momentum transfer form factor data [1,2], that mixing scheme is inadequate; it leads to inconsistencies with results from chiral perturbation theory [14,15] and is in conflict with $R_{J/\psi}$. An ansatz, however, where the four relevant wave functions are assumed to have the same form but different values at the origin, i.e. different decay constants, meets all requirements: It leads to very good results for the transition form factors, the decay widths for $\eta(\eta') \to \gamma\gamma$ and for $R_{J/\psi}$. The decay constants determined in this analysis are in good agreement with the recent results from chiral perturbation theory in which also the conventional octet-singlet mixing scheme is given up [16].

Another interesting problem that may be investigated within our approach, is the significance of intrinsic charm in the η and η' mesons. Recently a substantial charm component in the η' meson has been proposed in order to explain the large branching ratio of the decay $B \to K\eta'$ [17, 18]. Since the $\eta\gamma$ and $\eta'\gamma$ transition form factors at large Q^2 are sensitive to intrinsic charm, our analysis may shed further light onto the issue of the intrinsic charm magnitude.

The paper is organized as follows: First we present the proper expansion of pseudoscalar mesons in terms of parton Fock states and discuss properties of the light-cone wave functions associated with the light-quark valence Fock states (Sect. 2). In Sect. 3 we calculate the $\eta\gamma$ and $\eta'\gamma$ transition form factors. We employ the modified hard scattering approach (mHSA) in which the form factors are described by convolutions of perturbatively calculable hard scattering amplitudes and non-perturbative light-cone wave functions [19]. In contrast to the standard approach (sHSA) of Brodsky and Lepage [5], the transverse momenta of the partons and Sudakov suppressions are also taken into account in the mHSA. In this section we also discuss how to include the intrinsic charm contribution to the form factors. In Sect. 4 we present numerical results on the transition form factor obtained on the basis of the conventional octet-singlet mixing scheme. We are going to demonstrate that this mixing scheme seems to be inadequate. A more general mixing scheme with two mixing angles is discussed in Sect. 5. As it will turn out, this scheme leads to a very good description of the transition form factors. Also several other phenomenological constraints are satisfied within this scheme. The size of an eventual contribution from intrinsic charm is also estimated in this section. We end the paper with our conclusions (Sect. 6).

2 Fock states, light-cone wave functions and evolution

For the calculation of the transition form factors within the mHSA we need a parton Fock state decomposition of the mesons. Most generally, assuming isospin symmetry to be exact, we can write ($P = \eta, \eta'$)

$$\begin{aligned} |P\rangle \; = \; & \Psi_P^8 \, |u\bar{u} + d\bar{d} - 2s\bar{s}\rangle/\sqrt{6} \; + \\ & \Psi_P^1 \, |u\bar{u} + d\bar{d} + s\bar{s}\rangle/\sqrt{3} \; + \\ & \Psi_P^g \, |gg\rangle \; + \; \Psi_P^c \, |c\bar{c}\rangle \; + \ldots \end{aligned} \tag{1}$$

where the light quarks are arranged in terms of the $SU(3)_F$ octet and singlet combinations. This choice is convenient but not mandatory. For instance, a basis where the $|u\bar{u} + d\bar{d}\rangle$ and $|s\bar{s}\rangle$ parts are treated separately is a reasonable choice, too. In (1) we also allow for gluon and charm components that may appear due to the $U(1)_A$ anomaly. The ellipses stand for higher Fock states with additional gluons and/or $q\bar{q}$ pairs. Their contributions to the transition form factors are suppressed by powers of α_s/Q^2 [5], where α_s is the strong coupling constant, and will therefore be neglected in our analysis.

Following [8,20], we write the wave functions associated with the light-quark Fock states as ($i = 8, 1$)

$$\Psi_P^i(x, \mathbf{k}_\perp) := \frac{f_P^i}{2\sqrt{6}} \, \phi_P^i(x) \, \Sigma_P^i(\mathbf{k}_\perp/\sqrt{x\bar{x}}) \; . \tag{2}$$

Here, the momentum fraction x and the transverse momentum $\mathbf{k}_\perp$ refer to the quark; the antiquark momentum is characterized by $\bar{x} = 1 - x$ and $-\mathbf{k}_\perp$. The transverse momentum part, Σ_P^i, of the wave function is normalized as

$$\int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3} \, \Sigma_P^i(\mathbf{k}_\perp/\sqrt{x\bar{x}}) = 1 \; . \tag{3}$$

f_P^i is the decay constant of the pseudoscalar meson P through the Fock state i. In the hard scattering approach, f_P^i is related to the value of the corresponding wave function at the origin of the configuration space

$$\int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3} \int_0^1 \mathrm{d}x \, \Psi_P^i(x, \mathbf{k}_\perp) = \frac{f_P^i}{2\sqrt{6}} \; . \tag{4}$$

Explicit parameterizations of the charm and gluon wave functions are not needed in our analysis. The weak matrix elements that define the decay constants read

$$\langle 0 | J_{\mu 5}^i | P(p) \rangle = \imath \, f_P^i \, p_\mu \tag{5}$$

where the axial vector currents are given by

$$\begin{aligned} J_{\mu 5}^8 &= \frac{1}{\sqrt{6}} \left(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\,\bar{s}\gamma_\mu\gamma_5 s \right) \; , \\ J_{\mu 5}^1 &= \frac{1}{\sqrt{3}} \left(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s \right) \; . \end{aligned} \tag{6}$$

The definition of the decay constants (5) also applies to the charm component ($i = c$) with the current $J_{\mu 5}^c = \bar{c}\gamma_\mu\gamma_5 c$.

The distribution amplitudes for the octet components $\phi^8_P(x)$ have the same expansion upon Gegenbauer polynomials $C_n^{(3/2)}$ as the pion one [5],

$$\phi^8_P(x) = 6\,x\,\bar{x}\,\left\{1 + \sum_{n=2,4,\ldots} B^8_{Pn}(\mu_F)\; C_n^{(3/2)}(2x-1)\right\}\,. \tag{7}$$

The non-perturbative expansion coefficients evolve with the factorization scale $\mu_F(\propto Q^2)$ as

$$B^8_{Pn}(\mu_F) = B^8_{Pn}(\mu_0)\,\left\{\frac{\alpha_s(\mu_F)}{\alpha_s(\mu_0)}\right\}^{\gamma_n}\,. \tag{8}$$

Here, μ_0 is a typical hadronic scale of reference for which we choose a value of 0.5 GeV. Since the anomalous dimensions γ_n are positive fractional numbers increasing with n, all distribution amplitudes evolve into $\phi_{AS}(x) = 6\,x\,\bar{x}$ asymptotically. Rather similar forms of the octet and pion distribution amplitudes are to be expected from symmetry considerations. Since, to a very good approximation, the latter distribution amplitude equals the asymptotic form[2] this should be the case for ϕ^8_P, too. To deal with eventual small deviations from the asymptotic form it is sufficient to consider only the first non-trivial contribution $B^8_{P2} \neq 0$ with $C_2^{(3/2)}(z) = 3/2\,(5\,z^2-1)$ and $\gamma_2 = 50/81$.

In the singlet case evolution is more complicated. The evolution equation involves an anomalous dimension matrix which mixes the singlet and the two-gluon distribution amplitudes. In [21] the eigenfunctions and eigenvalues of the evolution equation have been calculated. The results for three flavors read

$$\begin{aligned}\phi^1_P(x) &= 6x\bar{x}\left\{1 + \sum_{n=2,4,\ldots}\left[B^1_{Pn}(\mu_F) + \rho^g_n B^g_{Pn}(\mu_F)\right]\right.\\ &\qquad\left.\times\; C_n^{(3/2)}(2x-1)\right\}\\ \phi^g_P(x) &= (x\,\bar{x})^2 \sum_{n=2,4,\ldots}\left[\rho^1_n B^1_{Pn}(\mu_F) + B^g_{Pn}(\mu_F)\right]\\ &\qquad\times\; C_{n-1}^{(5/2)}(2x-1)\,.\end{aligned} \tag{9}$$

The indices 1 and g on the r.h.s. of this equation characterize the two eigenfunctions. A common factor $f^1_P/2\sqrt{6}$ is pulled out of both the distribution amplitudes, see (2). Since only Gegenbauer polynomials $C_n^{(5/2)}$ of odd order contribute to the the gluon distribution amplitude it possesses the properties $\phi^g_P(x) = -\phi^g_P(\bar{x})$ and $\int_0^1 \phi^g_P(x)\,\mathrm{d}x = 0$ while $\phi^i_P(x) = \phi^i_P(\bar{x})$ and $\int_0^1 \phi^i_P(x)\,\mathrm{d}x = 1$ for the light quarks. The interesting point is that once ϕ^1_P is determined, say from experiment, the corresponding gluon distribution amplitude is, in principle, fixed by evolution. This situation is quite similar to the one in deep inelastic lepton-hadron scattering. In particular, if $\phi^1_P = \phi_{\mathrm{AS}}$ then $\phi^g_P = 0$. For the case $n = 2$ we quote the numerical values of the anomalous dimensions $\gamma^{1,g}_n$, controlling the evolution of the singlet and gluon distribution amplitudes analogue to (8), and the coefficients $\rho^{1,g}_n$ in the eigenfunctions that are induced by the gluon/quark admixtures ($C_1^{(5/2)}(z) = 5\,z$):

$$\begin{aligned}\gamma^1_2 &= 0.59 \qquad \rho^1_2 = 1.42\\ \gamma^g_2 &= 1.24 \qquad \rho^g_2 = -0.025\,.\end{aligned} \tag{10}$$

Finally, following [8,20,22], the transverse shape of the wave function is chosen to be a simple Gaussian ($i = 1, 8$)

$$\Sigma^i_P(\mathbf{k}_\perp/\sqrt{x\bar{x}}) = \frac{16\pi^2\,(a^i_P)^2}{x\,\bar{x}}\,\exp\left[-\frac{(a^i_P)^2\,\mathbf{k}_\perp^2}{x\,\bar{x}}\right]\,. \tag{11}$$

In the case of the pion the transverse size parameter a_π is fixed through the constraint [22]

$$\int \mathrm{d}x\; \Psi_\pi(x,0) \;=\; \sqrt{6}/f_\pi\,. \tag{12}$$

That relation leads to the closed formula $a_\pi^{-2} = (1 + B_{\pi 2}(\mu_F))\,8\pi^2\,f_\pi^2$ under the assumption $B_{\pi n} = 0$ for $n > 2$. For the asymptotic distribution amplitude one obtains $a_\pi = 0.86\ \mathrm{GeV}^{-1}$ corresponding to a r.m.s. transverse momentum of 370 MeV. For simplicity, we assume $a^i_P = a_\pi$, $i = 1, 8$ throughout this work. The present data do not allow to detect differences between the individual transverse size parameters.

We note that, leaving aside the intrinsic gluon and charm components, the Fock state decomposition (1) already includes four independent wave functions that characterize the light-quark contributions to the η and η' mesons, each in principle with its own distribution amplitude, transverse shape function and value at the origin. It is of course a formidable task to determine all the parameters, that enter the wave functions Ψ^i_P, completely from phenomenological constraints. As already mentioned in the introduction one needs additional assumptions in order to simplify the analysis (see Sect. 4, 5).

3 Meson-photon transition form factor

We write the $P\gamma\gamma^*$-vertex (see Fig. 1) as

$$\Gamma_\mu(q_1^2 = 0, q_2^2 = -Q^2) = i\,e_0^2\,F_{P\gamma}(Q^2)\,\epsilon_{\mu\nu\kappa\lambda}\,p^\nu\,q_1^\kappa\,\varepsilon^\lambda\,.$$

Following [7,8], we calculate the $P\gamma$ transition form factors ($P = \eta, \eta'$) for $Q^2 \geq 1\ \mathrm{GeV}^2$ within the mHSA. In that approach the transverse momentum dependence of the hard scattering amplitude is retained, and Sudakov

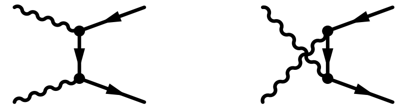

Fig. 2. Feynman graphs that determine the leading order hard scattering amplitude

[2] For comparison, within the modified HSA [7], a fit of $B_{\pi 2}$ to the CLEO data on the $\pi\gamma$ transition form factor [1] yields a value of -0.02 ± 0.1 at the scale μ_0

suppressions are taken into account in contrast to the sHSA. Each quark term in (1) gives rise to an additive contribution to the $P\gamma$ transition form factor which is represented by a convolution of the corresponding wave function, the hard scattering amplitude and a Sudakov factor ($i = 8, 1, c$):

$$F^i_{P\gamma}(Q^2) = \int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^2 b}{4\pi}\, \hat{\Psi}^i_P(x,\mathbf{b})\, \hat{T}^i_P(x,\mathbf{b},Q) \times \exp\left[-\mathcal{S}(x,\mathbf{b},Q)\right] \;, \quad (13)$$

where $\mathbf{b}$, canonically conjugated to the transverse momentum, is the quark-antiquark separation in the transverse configuration space. $\hat{\Psi}^i_P$ and $\hat{T}^i_P$ represent the Fourier transforms of the wave functions defined in (1) and the hard scattering amplitudes, respectively. To lowest order of perturbative QCD, the hard scattering amplitudes are to be calculated from the two Feynman graphs shown in Fig. 2. For the light-quark contributions ($i = 8, 1$) one finds

$$T^i_P(x,\mathbf{k}_\perp,Q) \;=\; \frac{2\sqrt{6}\, C_i}{x\, Q^2 + \mathbf{k}^2_\perp + x\,\bar{x}\, M^2_P} + (x \leftrightarrow \bar{x}) \quad (14)$$

in the transverse momentum space. The charge factors read $C_8 = (e^2_u + e^2_d - 2e^2_s)/\sqrt{6}$ and $C_1 = (e^2_u + e^2_d + e^2_s)/\sqrt{3}$. Since the masses M_P of the η and η' mesons are rather large we allow for corresponding corrections in the hard scattering amplitudes. Due to the symmetry of the distribution amplitudes under $x \leftrightarrow \bar{x}$ the two Feynman graphs provide identical contributions. The Fourier transformed amplitudes are proportional to $K_0(\sqrt{xQ^2 + x\bar{x}\, M^2_P}\, b)$ where K_0 is the modified Bessel function of order zero.

The Sudakov factor $\exp[-\mathcal{S}(x,b,Q)]$ takes into account gluonic corrections not accounted for in the QCD evolution of the wave functions. In the Sudakov factor b plays the role of an infrared cut-off; it sets up the interface between the non-perturbative soft gluon contributions – still contained in the hadronic wave function – and perturbative soft gluon contributions accounted for by the Sudakov factor. The gliding factorization scale to be used in the evolution of the wave functions is, hence, chosen to be $\mu_F = 1/b$. The Sudakov factor has been calculated by Botts and Sterman [19] in next-to-leading-log approximation. The explicit form of the Sudakov function which has been slightly improved, can, for instance, be found in [23].

The charm contribution to the $P\gamma$ transition form factor can be estimated in close analogy to the calculation of the $\eta_c\gamma$ form factor [24]. An important difference to the light-quark case consists in the mass of the charm quark ($m_c \simeq 1.5$ GeV) that provides a second large scale in the process. The hard scattering amplitude is therefore to be modified accordingly. The distribution amplitude ϕ^c_P is expected to behave similar to the η_c distribution amplitude and should, in particular, exhibit a pronounced maximum at $x = 1/2$. It therefore suffices to use the peaking approximation $\phi^c = \delta(x - 1/2)$. Including transverse momentum corrections to $O(\mathbf{k}^2_\perp/m^2_c)$, one finds for the charm contribution the reasonable approximation

$$F^c_{P\gamma}(Q^2) \;=\; \frac{4\, e^2_c\, f^c_P}{Q^2 + M^2_P/2 + 2m^2_c + 2\langle \mathbf{k}^2_\perp\rangle_c} \;, \quad (15)$$

where, according to [24], a value of 710 MeV is used for the r.m.s. transverse momentum of the charm quarks. Details of this approximation and an assessment of its quality can be found in [24]. Equation (15) possesses the highly welcome feature that, except of the decay constants and the r.m.s. transverse momentum, no further details of the charm wave function are required. Furthermore it rather represents an underestimate of the intrinsic charm contribution: Going beyond the peaking approximation and/or inserting a value of the r.m.s. transverse momentum closer to the value of 370 MeV that we use for the light-quark components, would even increase the magnitude of the charm contribution.

The two-gluon components of the η and η' mesons play no direct role in the analysis of the transition form factors since their coupling to photons is suppressed by the strong coupling constant α_s. Moreover, the formation of a pseudoscalar meson from two vector particles requires orbital angular momentum. This implies a factor $\mathbf{k}_\perp$ in the corresponding spin-wave function that leads to an additional suppression factor $\mathbf{k}^2_\perp/Q^2$ of the two-gluon contributions.

It is instructive to consider the asymptotic behavior of the transition form factors. For $\ln Q^2 \to \infty$ the Sudakov factor damps any contribution to the form factors except those from configurations with small quark-antiquark separations. Contributions from such configurations are actually considered in the sHSA. Hence, the mHSA and the sHSA have the same asymptotic limit. Since, for $\ln Q^2 \to \infty$, any distribution amplitude evolves into the asymptotic one, the limiting behavior of the transition form factors (for $f^c_P = 0$)

$$Q^2\, F_{P\gamma}(Q^2) \;\overset{\ln Q^2 \to \infty}{\longrightarrow}\; \sqrt{\frac{2}{3}}\, f^8_P + \frac{4}{\sqrt{3}}\, f^1_P \;. \quad (16)$$

is model-independent.

4 The octet-singlet mixing scheme

The most obvious way to reduce the number of parameters and to make contact to phenomenology, is to simplify the general ansatz (1) by adopting the usual $SU(3)_F$ octet-singlet mixing scheme. Corresponding Fock state components of the η and η' mesons are then controlled by one and the same wave function. The octet-singlet mixing scheme is defined through the relations

$$\begin{aligned} \Psi^8_\eta &= \;\;\Psi_8\, \cos\theta_P \;, \quad \Psi^8_{\eta'} = \Psi_8\, \sin\theta_P \;, \\ \Psi^i_\eta &= -\Psi_i\, \sin\theta_P \;, \quad \Psi^i_{\eta'} = \Psi_i\, \cos\theta_P \;, \quad (i = 1, g, c) \end{aligned} \quad (17)$$

for the valence Fock state wave functions defined in (1). Equation (17) implies analogous relations between the decay constants. For the light-quark wave functions, Ψ_8 and

Ψ_1, we use the ansatz (2). All the properties of light-quark wave functions discussed in Sect. 2 are valid for Ψ_8 and Ψ_1, too. The octet-singlet mixing scheme is based on the concept of octet (η_8) and singlet (η_1) mesons as $SU(3)_F$ basis states from which, by a unitary transformation, the physical mesons arise. This concept implies that Fock state components with singlet quantum numbers, in other words all Fock state components of the η_1 meson, contribute to the η and η' mesons through the relations in the second line of (17).

Approximate $SU(3)_F$ symmetry tells us that the octet and the pion wave functions cannot differ much from each other. Due to the larger quark masses involved, the octet distribution amplitude may, at the most, be slightly more midpoint-concentrated, i.e. $B_2^8 < 0$, than the pion one which is well described by the asymptotic form [7]. The singlet wave function is not related to the pion one by symmetry. Since the binding mechanisms of the quarks in the flavor octet and singlet channels are however similar, we expect the light-quark singlet wave function Ψ_1 to be not too different from that of the pion. Thus, a reasonable starting point of the analysis of transition form factors is the assumption $B_n^i = 0$, $a_i = a_\pi$ ($i = 1, 8$; $n \geq 2$). This ansatz coincides with the one used in [8]. There are still three parameters to be determined, the two decay constants f_1, f_8 and the mixing angle. Admittedly, additional information is required for this task since the two transition form factors do not suffice to fix these three parameters; for any value of the mixing angle an acceptable fit to the data can be obtained.

A useful constraint is provided by the two-photon decays of the η and η' mesons. Generalizing the PCAC result for the $\pi^0 \to \gamma\gamma$ (which is responsible for the constraint (12)), one assumes that the axial vector currents can be related via PCAC to the η and η' fields (see e.g. [25])

$$\begin{aligned} \partial^\mu J^8_{\mu 5}(z) &= f_\eta^8 \, M_\eta^2 \, \eta(z) + f_{\eta'}^8 \, M_{\eta'}^2 \, \eta'(z) + \dots \\ \partial^\mu J^1_{\mu 5}(z) &= f_\eta^1 \, M_\eta^2 \, \eta(z) + f_{\eta'}^1 \, M_{\eta'}^2 \, \eta'(z) + \dots \end{aligned} \qquad (18)$$

This leads to

$$\begin{aligned} \Gamma[\eta \to \gamma\gamma] &= \frac{9\alpha^2}{16\pi^3} \, M_\eta^3 \left[\frac{C_8 \, f_{\eta'}^1 - C_1 \, f_{\eta'}^8}{f_{\eta'}^1 \, f_\eta^8 - f_{\eta'}^8 \, f_\eta^1} \right]^2 , \\ \Gamma[\eta' \to \gamma\gamma] &= \frac{9\alpha^2}{16\pi^3} \, M_{\eta'}^3 \left[\frac{-C_8 \, f_\eta^1 + C_1 \, f_\eta^8}{f_{\eta'}^1 \, f_\eta^8 - f_{\eta'}^8 \, f_\eta^1} \right]^2 . \end{aligned} \qquad (19)$$

The various decay constants appearing in (19) can be expressed in terms of f_1, f_8 and θ_P by means of (17). The experimental values of the two-photon decay widths are [26]

$$\begin{aligned} \Gamma[\eta \to \gamma\gamma] &= (0.51 \pm 0.026) \text{ keV} , \\ \Gamma[\eta' \to \gamma\gamma] &= (4.26 \pm 0.19) \text{ keV} . \end{aligned} \qquad (20)$$

We do not include the value 0.324 ± 0.046 keV obtained from the Primakoff production measurement for $\eta \to \gamma\gamma$.

Using the asymptotic distribution amplitudes as well as universal transverse size parameters and ignoring an eventual charm contribution, we fit the decay constants f_1 and f_8 and the mixing angle to the transition form factor data above 1 GeV^2 [1,2,27,28] and the two-photon decay widths. The results of this excellent fit are shown in Table 1 and Fig. 3. Note the strong deviation from the dimensional counting behavior, $Q^2 \, F_{P\gamma} \simeq$ const., at small Q^2 in our approach. This is due to the inclusion of transverse momenta which lead to power corrections ($1/Q^{2n}$). In this respect the mHSA differs clearly from the sHSA. It is interesting to compare our values for f_1, f_8 and θ_P with those obtained from chiral perturbation theory (ChPT) [14,15]

$$\theta_P \simeq -20^\circ \qquad f_8 = 1.28 \, f_\pi \qquad f_1 \simeq 1.1 \, f_\pi \, . \qquad (21)$$

Whereas f_8 is theoretically on sound grounds by its relation to the pion and kaon decay constants

$$f_8 \, = \, \sqrt{\frac{4}{3} \, f_K^2 - \frac{1}{3} \, f_\pi^2} = 1.28 \, f_\pi \ , \qquad (22)$$

the other two parameters are subject to rather large phenomenological and theoretical uncertainties. For instance, f_1 may acquire scale dependent corrections of order $1/N_c$ due to the gluon anomaly [16]. Our fitted set of parameters, which is rather similar to that one quoted in [8], differs from (21) in the values of the mixing angle and the octet decay constant. The latter discrepancy is rather serious since, as we said, f_8 is well determined from ChPT (22). In phenomenological analyses based on the octet-singlet mixing scheme frequently (e.g. [8,29,30]), but not always (e.g. [31]), values for the mixing angle are obtained that are smaller in modulus than the ChPT value.

There is another phenomenological test of our parameters. According to [32], the radiative $J/\psi \to P\gamma$ decays are dominated by non-perturbative gluonic matrix elements:

$$R_{J/\psi} \, = \, \frac{\Gamma[J/\psi \to \eta'\gamma]}{\Gamma[J/\psi \to \eta\gamma]} \, \simeq \, \left| \frac{\langle 0|G\tilde{G}|\eta'\rangle}{\langle 0|G\tilde{G}|\eta\rangle} \right|^2 \left(\frac{k_{\eta'}}{k_\eta} \right)^3 \qquad (23)$$

where $k_P \, = \, M_{J/\psi} \, (1 - M_P^2/M_{J/\psi}^2)/2$ being the three-momentum of the P meson in the rest frame of the decaying J/ψ meson (with mass $M_{J/\psi}$). G is the gluonic field-strength tensor and $\tilde{G}$ its dual. We stress that these gluonic matrix elements are not related to the two-gluon components of the η and η' mesons appearing in (1) and (17). The light-quark contributions to these decays, while responsible for the $J/\psi \to \pi\gamma$ decay, are negligible small. The connection between the decay widths $\Gamma[J/\psi \to \eta(\eta')\gamma]$ and the gluonic matrix element, although heavily used in many analyses, has been occasionally criticized [33]. We only use a ratio of widths where most (or part) of the uncertainties may cancel. However, in account of the possible remaining uncertainties in (23) we do not include $R_{J/\psi}$ in our fit, but merely use it as an additional cross-check. Since the gluonic contributions have singlet quantum numbers, (23) reduces to

$$R_{J/\psi} \, = \, \cot^2 \theta_P \, \left(\frac{k_{\eta'}}{k_\eta} \right)^3 \qquad (24)$$

Table 1. Results of the χ^2 fit to the $\eta\gamma$ and $\eta'\gamma$ transition form factors and the two-photon decay widths in the octet-singlet mixing scheme ($a_i = a_\pi$, $B_2^i = 0$, $f_P^c = 0$). For comparison we also show results obtained from the parameter set OSS (see text)

	θ_P	f_8/f_π	f_1/f_π	χ^2/dof	$\Gamma_{\eta\to\gamma\gamma}$	$\Gamma_{\eta'\to\gamma\gamma}$	$R_{J/\psi}$
FIT	–15.1°	0.91	1.14	26/33	0.50 keV	4.10 keV	11.2
OSS	–22.2°	1.28	1.07	238/34	0.51 keV	4.26 keV	4.9

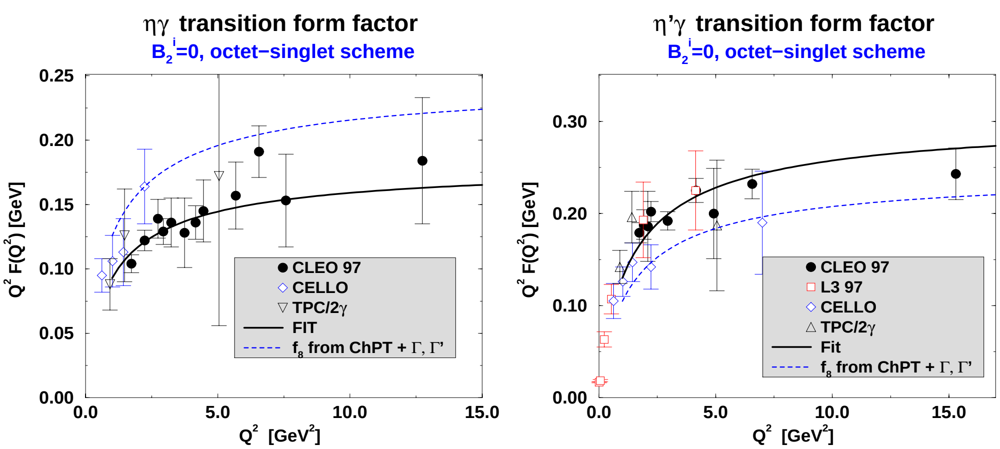

Fig. 3. Results for the $\eta\gamma$ and $\eta'\gamma$ transition form factors in the octet-singlet scheme ($a_i = a_\pi$, $f_P^c = 0$). For comparison we also show results obtained from the parameter set OSS. Data are taken from [1,2,27,28]

within the octet-singlet mixing scheme. Quite generally, the relation (24) may be regarded as an immediate consequence of singlet dominance and is, in so far, independent of (23). As inspection of Table 1 reveals, a value of $-15°$ for the mixing angle is in conflict with the experimental value of 5.0 ± 0.8 [26] for $R_{J/\psi}$.

The conflict between ChPT and the form factor analysis may be further elucidated by the following test: Keeping f_8 fixed at the ChPT value (22), we can determine θ_P and f_1 from the two-photon widths (19). The resulting set of values, termed OSS, is listed in Table 1. It is qualitatively and quantitatively equivalent to the parameter set (21). Using the set OSS, we evaluate the transition form factors again and arrive at very bad results (see Table 1 and Fig. 3). The ratio $R_{J/\psi}$, on the other hand, acquires a reasonable value.

One may hold the use of the asymptotic distribution amplitudes responsible for the apparent discrepancy between ChPT and the form factor analysis. In order to investigate this possibility we take the OSS set of parameters, keep $a_i = a_\pi$ as before and fit the two expansion coefficients B_2^i $(i = 8, 1)$ to the transition form factor data. We find $B_2^8(\mu_0) = -0.86$ and $B_2^1(\mu_0) = 0.13$. Thus, the demand of the ChPT values (21 or OSS) for the decay constants and the mixing angle still leads to a good fit to the transition form factors, the two-photon decay widths and $R_{J/\psi}$, but at the expense of an octet distribution amplitude which is very different from the asymptotic one. Such a strong modification of $\phi_8(x)$ seems unlikely, considering the quality of $SU(3)_F$ symmetry. It is also at variance with the recent estimate of B_2^8 ($\simeq -0.04$) obtained in the analysis of $\chi_{cJ} \to \eta\eta$ decays ($J = 0, 2$) [11]. We do not vary the values of the transverse size parameters since their influence on the transition form factor is rather small. They merely influence the curvature of the transition form factors at smaller values of momentum transfer.

From these considerations we conclude that the octet-singlet scheme for the η-η' system, although quite attractive due to its simplicity in phenomenological analyses, seems to be inadequate and should perhaps be given up in favor of a more general description of the η-η' system. We are going to investigate such a scheme in the next section.

5 The two-angle mixing scheme

In order to allow for four independent light quark decay constants (i.e. four independent wave functions at the origin of configuration space) we define now a new mixing scheme through the relations

$$\begin{aligned} \Psi_\eta^8 &= \Psi_8 \cos\theta_8\,, & \Psi_{\eta'}^8 &= \Psi_8 \sin\theta_8\ , \\ \Psi_\eta^i &= -\Psi_i \sin\theta_1\ , & \Psi_{\eta'}^i &= \Psi_i \cos\theta_1\ , \quad (i = 1, g, c)\ . \end{aligned} \tag{25}$$

Note that in contrast to (17) the scheme (25) does not make use of the concept of octet and singlet mesons as $SU(3)_F$ basis states. The analogous relations of the light-quark decay constants f_P^i have been introduced by Leutwyler [16] and, in a somewhat different parameterization, by Kiselev and Petrov [25].

Again, we are using the asymptotic distribution amplitudes and the universal transverse size parameters in the analysis of the transition form factors. Since we now have to determine one more parameter we need one more constraint. Thus, besides the two-photon decay widths (19),

Table 2. Results of the χ^2 fit to the $\eta\gamma$ and $\eta'\gamma$ transition form factors and the two-photon widths in the two-angle mixing scheme ($a_i = a_\pi$, $B_2^i = 0$, $f_P^c = 0$). Underlined parameters are kept fixed in the fit. For comparison we also show results obtained from the parameter set TAS (see text)

	θ_8	θ_1	f_8/f_π	f_1/f_π	χ^2/dof	$\Gamma_{\eta\to\gamma\gamma}$	$\Gamma_{\eta'\to\gamma\gamma}$	$R_{J/\psi}$
FIT	–22.2°	–9.1°	$\underline{1.28}$	1.20	26/33	0.50 keV	4.11 keV	5.1
TAS	$\underline{-22.2°}$	$\underline{-5.9°}$	$\underline{1.28}$	$\underline{1.22}$	41/34	0.51 keV	4.26 keV	6.2

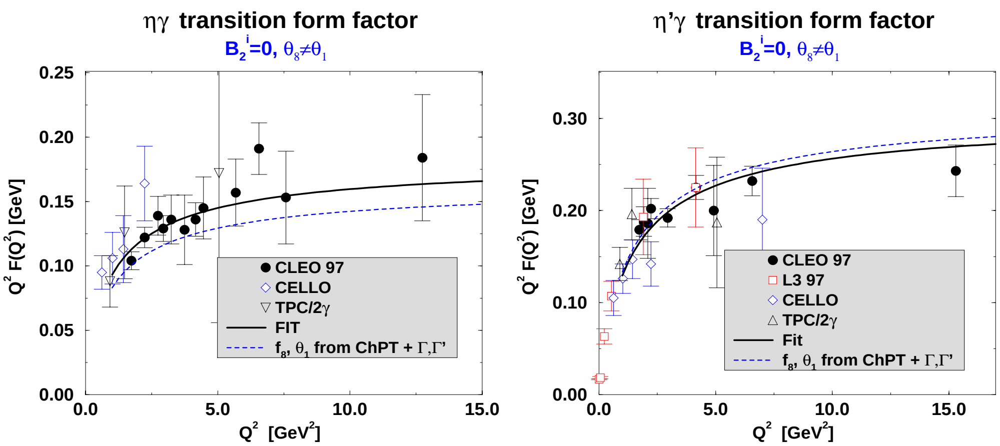

Fig. 4. Results for the $\eta\gamma$ and $\eta'\gamma$ transition form factors in the two-angle mixing scheme ($a_i = a_\pi$, $f_P^c = 0$). For comparison we also show results obtained from the parameter set TAS. Data are taken from [1,2, 27,28]

we use the theoretically reliable ChPT relation (22), i.e. we take $f_8 = 1.28\, f_\pi$. The results of that fit are shown in Table 2 and Fig. 4. As for the octet-singlet mixing scheme the results are very good and agree now quite well with a recent determination of these parameters from ChPT within the new mixing scheme [16]:

$$\begin{aligned} \theta_8 &= -20.5^\circ\ , \quad \theta_1 \simeq -4^\circ\ , \\ f_8 &= 1.28\, f_\pi\ , \quad f_1 \simeq 1.25\, f_\pi\ . \end{aligned} \tag{26}$$

The only noticeable deviation between (26) and the fitted parameters is to be observed for the angle θ_1 which, within ChPT, is related to the $SU(3)_F$ breaking effects in the decay constants of the pseudoscalar octet [16]

$$\sin(\theta_1 - \theta_8) \ \simeq\ \frac{2\sqrt{2}\,(f_K^2 - f_\pi^2)}{3\, f_8^2}\ . \tag{27}$$

This approximately valid relation leads to a difference $\theta_1 - \theta_8$ of the mixing angles of about 16°, larger but, in regard to the uncertainties in (27), not in conflict with the the fitted value of 13°.

The ratio of the $J/\psi \to P\gamma$ decay widths can still be cast into the form (24), but the angle appearing there is now to be understood as the mixing angle of the non-perturbative gluon contribution which may – and should – differ from the angle θ_1 defined in (25). In [25,30] this angle, and hence $R_{J/\psi}$, has been estimated using PCAC and taking into account a substantial non-zero strange quark mass (whereas $m_u, m_d \simeq 0$). This is well in the spirit of the new mixing scheme (25) where large $SU(3)_F$ breaking effects induce the substantial difference between the two mixing angles θ_8 and θ_1 (27). In our notation the result of [25] reads

$$R_{J/\psi} \ =\ \left| \frac{M_{\eta'}^2\,(f_{\eta'}^8 + \sqrt{2}\, f_{\eta'}^1)}{M_\eta^2\,(f_\eta^8 + \sqrt{2}\, f_\eta^1)} \right|^2 \left(\frac{k_{\eta'}}{k_\eta} \right)^3 \tag{28}$$

Inserting the fitted values of the parameters quoted in Table 2 into (28), one obtains a value of 5.1 for $R_{J/\psi}$ in perfect agreement with experiment. For comparison, we also show results in Table 2 and in Fig. 4 that are evaluated with a set of parameters determined from the two ChPT relations (22) and (27) as well as from the two-photon decay widths (19). This set of parameters, termed TAS, is, not surprisingly, close to the fitted values as well as to the ChPT values (26). It leads to a somewhat worse fit but is not in severe disagreement with the transition form factor data[3]. Also the value of $R_{J/\psi}$ is only about one standard deviation above the experimental result. The agreement with the transition form factor data can be improved in this case by allowing for non-zero Gegenbauer coefficients. Values of $B_2^8(\mu_0) \simeq 0.25$ and $B_2^1(\mu_0) \simeq 0$ lead to a reasonable fit of the data.

Recently, substantial intrinsic charm in the η' meson has been proposed [17,18] in order to explain the large branching ratios of the decays $B \to K\eta'$ and $B \to X_s\eta'$. From the experimental measurements the authors of [17] obtain an absolute value of 140 MeV for the charm decay constant $f_{\eta'}^c$ (see (5)). This surprisingly large value is claimed to be justified within a QCD sum rule analysis. Another analysis of B decays [18] yields the more moderate value of -50 MeV for $f_{\eta'}^c$. If $f_{\eta'}^c$ is that large, the

[3] The set of parameters quoted in [25] differs from the sets (26), TAS and the fitted one (see Table 2) substantially and is not consistent with the $\eta\gamma$ and $\eta'\gamma$ transition form factors

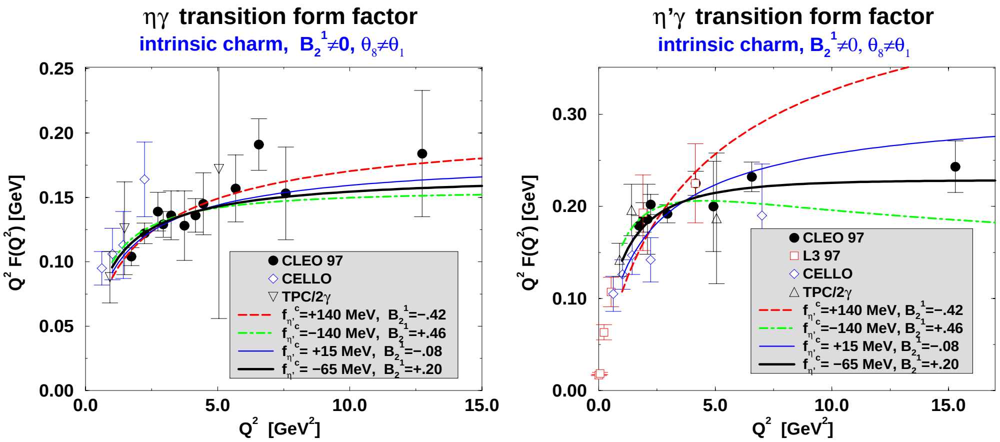

Fig. 5. Sample results for the $\eta\gamma$ and $\eta'\gamma$ transition form factors obtained from light quark (with the parameter set FIT quoted in Table 2, $B_2^8 = 0$ and $a_i = a_\pi$) and charm contributions. Data are taken from [1,2, 27,28]

radiative decay $J/\psi \to \eta'\gamma$ may be dominated by a contribution where the $c\bar{c}$ pair runs from the J/ψ to the η' meson instead of being annihilated. On that supposition the width of that process can be calculated along the same lines as that one for the $J/\psi \to \eta_c\gamma$ decay. The ratio of the two decay widths reads

$$\frac{\Gamma[J/\psi \to \eta'\gamma]}{\Gamma[J/\psi \to \eta_c\gamma]} = \kappa^2 \left(\frac{f_{\eta'}^c}{f_{\eta_c}}\right)^2 \left(\frac{k_{\eta'}}{k_{\eta_c}}\right)^3 \qquad (29)$$

in analogy to (24). κ represents the ratio of the J/ψ-η' and J/ψ-η_c wave function overlaps. In [34] κ was assumed to be unity, i.e. both the η' and the η_c mesons basically behave like non-relativistic bound states of heavy quarks with about the same overlap with the J/ψ wave function (each close to unity). From the experimental values of the decay widths one then estimates $|f_{\eta'}^c| = 6$ MeV in contradiction to the initial assumption. In regard to the large binding energy required for the $c\bar{c}$ component of the η' meson, a value of κ significantly less than unity seems not implausible. Consequently, a value larger than 6 MeV for $f_{\eta'}^c$ cannot really be excluded by means of (29).

We are now going to estimate the size of the intrinsic charm. Since the effect of the charm component of the η meson is suppressed by the small singlet mixing angle ($f_\eta^c = -\tan\theta_1\, f_{\eta'}^c$, see (25)) we can mainly concentrate ourselves on the $\eta'\gamma$ transition form factor in the following discussion. The large charm quark mass effectuates a strong suppression of the charm contribution to the $\eta'\gamma$ form factor at small values of Q^2 (see (15)); the main effect of it shows up for, say, $Q^2 \gtrsim 4$ GeV^2. The charm contribution therefore approaches its asymptotic behavior with a much slower rate than the light-quark contributions. This difference in the curvature is the crucial point that allows to disentangle the charm and the light-quark contributions. In order to determine the range of allowed $f_{\eta'}^c$ values, we fit this parameter as well as B_2^1 to the $\eta'\gamma$ and $\eta\gamma$ form factors, keeping the parameter set FIT given in Table 2 fixed. f_η^c is determined by means of (25). The variation of B_2^1 changes the strength of the singlet part of the light-quark contributions, that is dominant in the, for these considerations, most important η' case, and thus makes space for a charm contribution. We could have freed f_1 instead of B_2^1, but this procedure would have the disadvantage of eventually destroying the agreement of our results with ChPT and the good description of the two-photon decay widths. The numerical analysis yields the following range of allowed values for $f_{\eta'}^c$

$$-65\,\text{MeV} \le f_{\eta'}^c \le 15\,\text{MeV} \qquad (30)$$

The corresponding changes of B_2^1 are moderate (see Fig. 5) and do not lead to implausible singlet distribution amplitudes. The results for the form factors obtained with the values 15 and -65 MeV for $f_{\eta'}^c$ are shown in Fig. 5. For comparison, results with ± 140 MeV are also shown in this figure. The latter two values, which require drastic changes of B_2^1, lead to results for the $\eta'\gamma$ transition form factor in clear conflict with the data above 4 GeV^2. More restrictive bounds on $f_{\eta'}^c$ than (30) require more form factor data above 4 GeV^2. We stress that the approximation (15) rather underestimates the effect of intrinsic charm (see Sect. 3). Finally, we comment on $R_{J/\psi}$ and emphasize that this quantity is not included in our fits; it is only used as an accompanying test of the results. With the large range (30) of allowed $f_{\eta'}^c$ values the gluon dominance, assumed up to now for the $J/\psi \to \eta\gamma$ and $J/\psi \to \eta'\gamma$ decays, may not be true anymore, and, hence, the mentioned successful test is perhaps accidental. Whether the $J/\psi \to \eta(\eta')\gamma$ decay can reliably be explained by intrinsic charm in the $\eta(\eta')$ meson remains to be shown.

6 Conclusions

The modified hard scattering approach is shown to provide a consistent description of the $\eta\gamma$ and $\eta'\gamma$ transition form factors over a wide range of the momentum transfer 1 GeV$^2 \le Q^2 \le 15$ GeV^2, where experimental data is now available. It is to be emphasized that this result is not trivial at all: The rather strong deviations from the dimensional counting behavior ($Q^2\, F_{P\gamma} \simeq$ const.) at small momentum transfer appear as a consequence of the transverse momentum dependence and the Sudakov suppressions included in the mHSA. We have included the two-photon decay widths of the η and η' mesons in the analysis in order to reduce the degrees of freedom. The ratio $R_{J/\psi}$ of the $J\psi \to \eta'\gamma$ and $J/\psi \to \eta\gamma$ decay widths is merely used as an additional test.

Our analysis of the transition form factors allows us to extract interesting information on the η and η' wave functions. Our starting point is the assumption that, like the pion, the light-quark components of the η and η' mesons are described by the asymptotic form of the distribution amplitude and a Gaussian transverse momentum dependence with a universal transverse size parameter, $a_i = a_\pi$ $(i = 1, 8)$. The wave functions at the origin of the configuration space, or the corresponding decay constants, are considered as free parameters to be determined from the analysis. We found hints at an inadequacy of the conventional octet-singlet scheme used to describe the mixing and the $SU(3)_F$ symmetry breaking in the η-η' system. The fit yields values of the mixing angle and the octet decay constant which substantially deviate from the ChPT values. Moreover, the fitted set of parameters, quoted in Table 1, does not pass the $R_{J/\psi}$ test. Agreement with ChPT can only be obtained at the expense of large deviations of the octet wave function from the pion one. With regard to the quality of $SU(3)_F$ symmetry this seems to be unrealistic.

In contrast to the conventional octet-singlet mixing scheme, the more general two-angle mixing scheme [16,25] meets all requirements: It provides a very good description of the transition form factors with the asymptotic form of the distribution amplitudes. The set of parameters

$$\begin{aligned} \theta_8 &= -22.2^\circ \,, \quad \theta_1 \simeq -9.1^\circ \,, \\ f_8 &= 1.28\, f_\pi \,, \quad f_1 \simeq 1.20\, f_\pi \end{aligned} \tag{31}$$

is in reasonable agreement with the recent ChPT results obtained within that new mixing scheme [16] and reproduces the two-photon decay widths of the η and η' mesons as well as the ratio $R_{J/\psi}$. The parameter set (31) implies the asymptotic behavior of the transition form factors $Q^2 F_{P\gamma} \to 184$ MeV and 306 MeV for the η and η' cases, respectively. Thus, numerically the $\eta\gamma$ and $\pi\gamma$ transition form factor have the same asymptotic values.

Finally, we have also investigated whether a large intrinsic charm component in the η and η' mesons is allowed by the transition form factor data. From our analysis we estimate the range of allowed $f^c_{\eta'}$ values to be: -65 MeV $\le f^c_{\eta'} \le 15$ MeV. Values as large as 140 MeV, as suggested in [17], seem to be excluded. More restrictive bounds on $f^c_{\eta'}$ require more transition form factor data above 4 GeV^2.

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References

1. CLEO collaboration, J. Gronberg et al., Phys. Rev. **D57** (1998) 33
2. L3 collaboration, M. Acciarri et al., (1997), preprint CERN-PPE/97-110, submitted to Phys. Lett. B
3. CLEO collaboration, V. Savinov et al., (1995), in proceedings of the PHOTON 95 workshop, Sheffield (1995), eds. D.J. Miller et al., World Scientific
4. T.F. Walsh, P. Zerwas, Nucl. Phys. **B41** (1972) 551
5. G.P. Lepage, S.J. Brodsky, Phys. Rev. **D22** (1980) 2157
6. M. Hayakawa, T. Kinoshita, Phys. Rev. **D57** (1998) 465
7. P. Kroll, M. Raulfs, Phys. Lett. **B387** (1996) 848
8. R. Jakob, P. Kroll, M. Raulfs, J. Phys. **G**22 (1996) 45
9. I.V. Musatov, A.V. Radyushkin, Phys. Rev. **D**56 (1997) 2713
10. S.J. Brodsky et al., Phys. Rev. **D57** (1998) 245
11. J. Bolz, P. Kroll, G.A. Schuler, Phys. Lett. **B392** (1997) 198; and hep-ph/9704378, to be published in Eur. Phys. J. C
12. V.V. Anisovich, D.I. Melikhov, V.A. Nikonov, Phys. Rev. **D55** (1997) 2918
13. L. Ametller, J. Bijnens, A. Bramon, F. Cornet, Phys. Rev. **D45** (1992) 986
14. J. Gasser, H. Leutwyler, Nucl. Phys. **B250** (1985) 465
15. J.F. Donoghue, B.R. Holstein, Y.C.R. Lin, Phys. Rev. Lett. **55** (1986) 2766
16. H. Leutwyler, (1997), hep-ph/9709408
17. I. Halperin, A. Zhitnitsky, Phys. Rev. **D56** (1997) 7247
18. H.Y. Cheng, B. Tseng, (1997), hep-ph/9707316
19. J. Botts, G. Sterman, Nucl. Phys. **B325** (1989) 62; H. Li, G. Sterman, Nucl. Phys. **B381** (1992) 129
20. R. Jakob, P. Kroll, Phys. Lett. **B315** (1993) 463
21. V.N. Baier, A.G. Grozin, Nucl. Phys. **B192** (1981) 476; M.A. Shifman, M.I. Vysotskii, Nucl. Phys. **B186** (1981) 475
22. S.J. Brodsky, T. Huang, G.P. Lepage, Particles, Fields 2, eds. Z.Capri, A.N. Kamal (Banff Summer Institute, 1983) 143
23. M. Dahm, R. Jakob, P. Kroll, Z. Phys. **C68** (1995) 595
24. T. Feldmann, P. Kroll, Phys. Lett. **B413** (1997) 410
25. A.V. Kiselev, V.A. Petrov, Z. Phys. **C58** (1993) 595
26. Particle Data Group, R.M. Barnett et al., Phys. Rev. **D54** (1996) 1
27. CELLO collaboration, H.J. Behrend et al., Z. Phys. **C49** (1991) 401
28. TPC/2γ collaboration, H. Aihara et al., Phys. Rev. Lett. **64** (1990) 172
29. A. Bramon, R. Escribano, M.D. Scadron, Phys. Lett. **B403** (1997) 339
30. P. Ball, J.M. Frere, M. Tytgat, Phys. Lett. **B365** (1996) 367
31. F.J. Gilman, R. Kauffman, Phys. Rev. **D36** (1987) 2761
32. V.A. Novikov, M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. **B165** (1980) 55
33. M. A. Luty, R. Sundrum, Phys. Lett. **B312** (1993) 205
34. A. Ali, C. Greub, (1997), hep-ph/9707251, submitted to Phys. Rev. D